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THE FILTERING OF A SUSPENSION OF SOLID PARTICLES†

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The changes in the conducting properties of a porous medium when a fluid with suspended solid particles moves through it are investigated. A model system of equations is presented, a class of exact solutions is obtained for initial problem and of the initial-boundary-value problem. A comparison is made with experiment using the example of the percolation of a suspension into a pure uniform sample. It is shown that the model considered describes, qualitatively correctly, the behaviour of the main parameters of the porous medium and the flow of the suspension during colmatage (silt deposition). © 1999 Elsevier Science Ltd. All rights reserved.

The phenomenon of the trapping by a porous medium of particles suspended in a fluid flow (colmatage) must be taken into account when solving many important practical problems. These include the problem of purifying liquids from impurities by filtering, the investigation of the deposition of clay particles from the filtrate of drilling fluid on the surface of an oil well and in the preborehole region of the layer when drilling gas and oil wells and a number of other problems [1–3]. The investigation of these phenomena is usually based on various approximate relations.

The model presented below is based on exact balance relations, it extends existing approaches to the investigation of the problem considered, and it takes into account the effect of the seepage rate on the kinetics of the colmatage and the presence of dead-end pores in the porous medium. Section 1 is devoted to deriving a system of differential equations for the changes in the porosity of the medium and the concentration of the solid phase of the suspension. In Section 2 solutions of simple form are described, which, for a certain choice of the parameters occurring in it, turn out to be asymptotic solutions of the initial-boundary-value problem. The exact solutions of the initial-value problem and the initial-boundary-value problem for a general form of initial and boundary data are constructed in Sections 3 and 4, respectively. The solutions have been well known for approximate or linearized equations. The model employed is compared with experiment in Section 5.

1. DESCRIPTION OF THE MODEL

Suppose c is the volume density of particles in the flow of the suspension, $\mathbf{v}$ is the seepage rate and m is the porosity of the medium, which are the required functions of the point $\mathbf{x}$ and the time t. The velocities of the suspended particles and of the fluid carrying them are assumed to be the same, and the particles themselves, the fluid and the porous medium in its initial state are assumed to be incompressible.

As time passes the suspended particles may remain in the flow or be retained in the porous medium, shutting off the entrances to the pores. In this case, if a pore has several entrances and exists, it remains accessible to the flow. Otherwise it is bound into the framework of the porous medium, extracting the particle from the flow and the corresponding fluid volume. The framework of the porous medium formed in this way remains incompressible. The contribution of such (dead-end) pores is characterized by the parameter $\varkappa$: the trapping of particles by each unit volume leads to the formation of $1 + \varkappa$ volumes of the framework of the porous medium. The value of this parameter is proportional to the volume fraction (in units of volume of the porous space) of dead-end pores.

Under these conditions the balance of the volume of the solid phase (in the suspension and retained by the porous medium) is written in the form

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$$\frac{d}{dt}\int_{\Omega}\left[mc+\frac{1}{1+\varkappa}(m_0-m)\right]dx=-\int_{\partial\Omega}c\mathbf{v}\cdot\mathbf{n}ds$$

where Ω is an arbitrarily fixed region of space, $\partial\Omega$ is its boundary with outward normal $\mathbf{n}$, and m_0 is the specified initial porosity distribution.

The volume balance of the fluid (in the flow and that cut off from it during colmatage) requires the following equation to be satisfied

$$\frac{d}{dt}\int_{\Omega}\left[m(1-c)+\frac{\varkappa}{1+\varkappa}(m_0-m)\right]dx=-\int_{\partial\Omega}(1-c)\mathbf{v}\cdot\mathbf{n}ds$$

In the region of a smooth change in the required functions, these integral relations are equivalent to the following system of differential equations

$$\partial(mc)/\partial t+\operatorname{div}\mathbf{v}c=(1+\varkappa)^{-1}\partial m/\partial t \tag{1.1}$$

$$\partial(m(1-c))/\partial t+\operatorname{div}\mathbf{v}(1-c)=\varkappa(1+\varkappa)^{-1}\partial m/\partial t \tag{1.2}$$

A consequence of these equations is the suspension-volume balance equation

$$\operatorname{div}\mathbf{v}=0 \tag{1.3}$$

Analogues of system (1.1), (1.2), at various approximation levels, are available only for the case when $\varkappa = 0$. Equations differing from (1.1) and (1.2) in having no terms with partial derivatives with respect to time on the left-hand sides have been used in [1]; Eq. (1.3) was assumed to be satisfied. An equation corresponding to (1.1) was written in [4, 5] in non-divergent form, but if Eq. (1.3) is used, an equation of the form (1.1) with a negative value of $\varkappa = (m/m_0 + 1)^{-1} - 1$ occurs. In the conditions of incompressibility, assumed in [6, 7], the solid-phase balance equation is identical with (1.1), but Eq. (1.3) contains an additional term $\partial m/\partial t$. Equation (1.1), written in different terms, was presented in [8] for the case when $\varkappa = 0$.

The equation of colmatage kinetics, which relates the rate of porosity change to the flow parameters, is often taken in the form [1, 6, 7]

$$\partial m/\partial t=-\gamma mc$$

where γ is a certain constant. The drawback of this scheme is that trapping of the particles always occurs when m and c are non-zero. Kinetics of a more general form

$$\partial P/\partial t=\alpha(A-P)c$$

have also been used [8], where P is the volume fraction (per unit of space volume) of particles trapped by the porous medium, A is its maximum possible value and α is a constant. The latter, in practice, depends on the seepage rate, since it is not the value of the concentration itself but the value of the whole particle flux $|\mathbf{v}c|$ which affects the colmatage intensity.

We will use the linear dependence of α on the modulus of the seepage rate and, taking into account the fact that, in reality, $P = m_0 - m$, we will write the colmatage kinetics equation in the form

$$\partial m/\partial t=-(1+\varkappa)\beta v\,c(m-m_*),\quad v=|\mathbf{v}| \tag{1.4}$$

The constant β, which has the dimension L^{-1}, is proportional to the number of trappings of a suspended particle by the porous medium, referred to the traversed path. The minimum possible value of the porosity m_* during colmatage is due to the presence of the connected system of pores, the diameter of which exceeds the dimensions of the suspended particles. Both β and m_* depend on the fractional properties of the suspension and the structure of the porous space, but no systematic investigations of this have been made.

We will assume that the flow of the suspension obeys Darcy's law [1, 6–8]

$$\mathbf{v}=-(k/\mu)\nabla p \tag{1.5}$$

with a suspension viscosity μ, a pressure in the suspension p and a permeability of the porous medium k, related to the porosity m by the power law

$$k = k_0(m/m_0)^n \tag{1.6}$$

Here k_0 and m_0 are the initial values of k and m respectively, while the exponent n lies in the range $2 < n < 4$ [9].

In order to compare the conclusions obtained using this model with experimental results, we will henceforth consider a special case of system (1.1)–(1.6). Namely, we will assume that $\varkappa = 0$, $\mu = \text{const}$, the flow is one-dimensional, and the x axis is directed along the flow. By relation (1.3) the seepage rate in this case depends only on time. We will also assume that $v = \text{const}$ and we will use Eqs (1.5) and (1.6) to establish how the pressure depends on x and t. Note that most of the discussion here remains true for the specified dependence $v(t)$ or $\varkappa > 0$.

Hence, below we will be concerned with the following system of equations

$$\begin{aligned} &m\partial c/\partial t + v\partial c/\partial x = -\beta vc(1-c)(m-m_*) \\ &\partial m/\partial t = -\beta vc(m-m_*) \end{aligned} \tag{1.7}$$

2. PARTICULAR SOLUTIONS

We will seek solutions of system (1.7) of the form

$$m = m(\xi),\ c = c(\xi),\ \xi = x - \sigma t \tag{2.1}$$

where σ is the required constant. Such solution are, firstly, fairly simple, and if the parameters m^* and c_* occurring in them are chosen in the required way, they become, as will be shown in Section 5, asymptotic solutions of the initial-boundary-value problem. Secondly, the states corresponding to them characterize, to a considerable extent, the internal properties of the model itself and the constant described, in which a certain equilibrium is established between the inflow of the particles with the flow of the suspension and their trapping by the porous medium.

As follows from (1.7) and (2.1), the functions $m(\xi)$ and $c(\xi)$ satisfy the system of equations

$$-\sigma mc' + vc' = -\beta vc(1-c)(m-m_*),\quad \sigma m' = \beta vc(m-m_*) \tag{2.2}$$

It follows from this that $\sigma m(1-c) + vc \equiv \sigma m^*$ (the right-hand side of this identity corresponds to the satisfaction of the conditions $c \to 0$, $m \to m_*$, $0 < m^* < 1$ as $\xi \to +\infty$). Hence, the concentration c is expressed in terms of m and σ in the form

$$c = (m^* - m)(v/\sigma - m)^{-1}$$

For the general case $c(-\infty) \neq 0$ the requirement that the solution of system (2.2) should "flatten out" as $\xi \to -\infty$ denotes that here $c \to c_*$ and $m \to m_*$, for a certain value of c_*, $0 < c_* < 1$. Hence, the propagation velocity σ of this wave must be defined by the following expression, which has a fairly simple physical meaning

$$\sigma = (vc_*)/(m^* - m_* + m_*c_*) \tag{2.3}$$

It follows from system (2.2) and the representation of the concentration in terms of the porosity and the velocity of the wave obtained earlier, that

$$(v/\sigma - m)(m - m_*)^{-1}(m^* - m)^{-1}m' = \beta v/\sigma$$

Solving this equation and using (2.3) we obtain

$$\frac{m - m_*}{m_1 - m_*}\left(\frac{m_* - m}{m^* - m_1}\right)^{c_* - 1} = \exp[\beta c_*(\xi - \xi_1)v/\sigma]$$

where m_1 is the porosity value for a certain $\xi = \xi_1$.

We will now introduce a function $s(\xi)$ instead of $m(\xi)$ by the rule

$$s = (m - m_*)/(m^* - m_*) \tag{2.4}$$

Assuming then that $\xi_1 = 0$ and choosing $s_1 = s(m_1)$ in a corresponding way we obtain the following equation for $s(\xi)$

$$s(1-s)^{c_*-1} = d(\xi) \equiv \exp(\beta c_* \xi \nu / \sigma), \quad 0 < s < 1 \tag{2.5}$$

Here

$$c = c_*(1-s)(1-c_* s)^{-1} \tag{2.6}$$

Solution (2.4)–(2.6) possesses one characteristic property, namely, it follows from (2.4), (2.2) and (2.6) that

$$s' = \beta \nu\, \sigma^{-1} c_* s(1-s)(1-c_* s)^{-1}$$

$$c'' = \beta \nu\, \sigma^{-1} c_*^2 (1-c_*)(c_* s^2 + 2(1-c_*)s - 1)(1-c_* s)^{-4} s'$$

Hence, the function $c(\xi)$ decreases monotonically and its graph will be convex upwards when $\xi < \xi_c$ and convex downwards when $\xi_c < \xi$ if ξ_c is such that

$$s(\xi_c) = \left(1 - c_* + \sqrt{(1-c_*)^2 + c_*}\right)^{-1}$$

This feature is also characteristic for the majority of experimental concentration distribution curves in the sample.

3. THE CAUCHY PROBLEM

An investigation of the initial-boundary-value problem corresponding to the experimental conditions first requires a consideration of the Cauchy problem—the establishment from the initial data

$$m(x, 0) = m_0(x), \quad c(x, 0) = c_0(x) \tag{3.1}$$

of the solution of system (1.7) in the half-plane $\{0 < t, -\infty < x < +\infty\}$. We will use the method of characteristics to construct the solution of the latter.

Suppose $s = \xi(t, x_0)$ is the characteristic of the first equation of system (1.7), emerging from the point $(x_0, 0)$ and arriving at the point (x, t). It is defined as the solution of the differential equation $dx/dt = v/m$ with the condition $\xi(0, x_0) = x_0$ and the required values of the porosity $m(x, x_0)$ on this curve. The latter can be obtained from the following considerations.

If $dm/dt \equiv g(t, x_0)$ on this curve, then, firstly, $g = \partial m/\partial t + v m^{-1} \partial m/\partial x$. Further, integration of the second equation of system (1.7) with respect to t for a fixed value of $x = \xi(t, x_0)$, shows that

$$m(x,t) = m_* + (m_0(x_0) - m_*)\exp\left(-\beta \nu \int_0^t c d\tau\right)$$

Hence, by Eq. (1.1)

$$g = [m_0'/(m_0 - m_*) + \beta m_0(1-c_0) - \beta m]\nu(m - m_*)/m$$

We now replace the parameter t along the characteristic by x and substitute the representation obtained for the function g into the right-hand side of the differential equation $dm/dx = mg/v$. Solving this equation and using initial conditions (3.1), we find that on the curve $x = \xi(t, x_0)$

$$m(x, x_0) = m_* + \Phi(x, x_0) \Big/ \left[1 + \beta \int_{x_0}^{x} \Phi(y, x_0) dy\right] \tag{3.2}$$

where

$$\Phi(x, x_0) = (m_0(x) - m_*)\exp\left(\beta \int_{x_0}^{x} [m_0(1-c_0) - m_*] dy\right) \tag{3.3}$$

To determine the values of the concentration on the characteristic we will use the fact that the differential relation $(c^{-1} + (1-c)^{-1}dc = -\beta(m - m_*)dx$ exists on it. Substituting the values obtained for the porosity into it, integrating and taking into account initial conditions (3.1) we conclude that when $x = \xi(t, x_0)$

$$c(x,x_0) = \left[1 + (c_0^{-1}(x_0) - 1)\exp\left(\beta\int\limits_{x_0}^{x}(m(y,x_0) - m_*)dy\right)\right]^{-1} \tag{3.4}$$

The equation of the characteristic itself, emerging from the point $(x_0, 0)$, can then be written in the form

$$ut = \int\limits_{x_0}^{x} m(y,x_0)dy \tag{3.5}$$

Omitting the details, we note that the solution of Cauchy problem (3.2)–(3.5) with the natural limits $0 < m_* \leqslant m_0(x)$, $0 < c_0(x) < 1$ on the initial data (3.1) will be defined over the whole half-plane $0 < t$. The equation of the characteristics (3.5) can be solved for x_0, while the solution of the problem can be represented in the form $m = m(x, t)$, $c = c(x, t)$. For continuous $m_0(x)$ and $c_0(x)$ the solution is continuous together with the derivative $\partial m/\partial t$. If $c_0(x)$ is a smooth function, $c(x, t)$ will also be smooth.

It can be shown that moving discontinuities of the solution only arise due to a discontinuity of the initial concentration values. On passing through them, as follows from the integral balances of the fluid- and solid-phase volumes, represented in Section 1, and the stability conditions, the seepage rate and the porosity retain their continuity. A discontinuity of this type is characteristic of the first equation of system (1.7). Fixed discontinuities are only possible when there is a sudden change in the function $m_0(x)$. On passing through them the seepage rate and the concentration remain continuous, but the construction of the corresponding solution requires investigations of the initial-boundary-value problem.

4. THE INITIAL-BOUNDARY-VALUE PROBLEM

For system (1.7) we will consider the problem with initial conditions (3.1), satisfied when $0 < x$, and the boundary condition

$$c(0, t) = c^0(t) \text{ when } 0 < t \tag{4.1}$$

The quadrant $\{0 < x, 0 < t\}$, in which we have to determine the solution, is split into two subregions: $\omega_+ = \{\xi(t, 0) < x, 0 < t\}$, where $x = \xi(t, 0)$ is the limiting characteristic, obtained from (3.2)–(3.5) when x_0 approaches zero there from the right; ω_- is the remaining part of the quadrant. In the region ω_+ a solution is uniquely defined by the initial conditions and is described by representations (3.2)–(3.5). When constructing the solution in the region ω_- we will use considerations analogous to those described in Section 3.

Suppose $\tilde{m}(x)$ is the porosity value on the curve $x = \xi(t, 0)$, obtained from (3.2) and (3.3) if we take $x_0 = 0$ there. On the characteristic $x = \xi(t, 0)$ of the first of the equations of system (1.7), emerging from the point $(0, t_0)$, the porosity m takes values $m(x, t_0)$, which, as in the Cauchy problem, satisfy the equation (we recall that d/dx denotes differentiation with respect to x along the characteristic)

$$dm/dx = (m - m_*)[g - \beta(m - m_*)], \quad g = m'/(\tilde{m} - m_*) + \beta(\tilde{m} - m_*)$$

Integration of this shows that, on this curve

$$\frac{1}{m - m_*} = \left[\frac{1}{m(0,t_0) - m_*} + \beta\int\limits_0^x \exp\left(\int\limits_0^y g(z)dz\right)dy\right]\exp\left(-\int\limits_0^x g(y)dy\right)$$

We will express the values of m and $m(0, t_0)$ occurring in this representation in terms of the initial and boundary data. To do this we first note that, according to Section 3, $m(x)$ is the solution of the differential equation

$$dm/dx = (m - m_*)\{m_0'/(m_0 - m_*) + \beta[m_0(1 - c_0) - m_*] - \beta(m - m_*)\}$$

which satisfies the initial condition $\tilde{m}(0) = m_0(0)$. Hence it follows that

$$g = m_0'/(m_0 - m_*) + \beta[m_0(1 - c_0) - m_*]$$

and hence the porosity values on the curve $x = \xi(t, t_0)$ can be represented in the form

$$m = m_* + h(x)\left[\frac{1}{m(0,t_0) - m_*} + \beta\int_0^x h(y)dy\right]^{-1}$$

$$h(x) = \Phi(x, 0)/(m_0(0) - m_*)$$

with $\Phi(x, x_0)$ from (3.3). We now integrate the second equation of system (1.7) with respect to t from $t = 0$ to $t = t_0$ with fixed $x = 0$. Using boundary condition (4.1) we conclude that

$$m(0,t_0) = m_* + (\tilde{m}(0) - m_*)\exp(-\beta v\int_0^{t_0} c^0(\tau)d\tau)$$

Substituting these values into the representation for the porosity obtained above, we find that on the characteristic $x = \xi(t, t_0)$

$$m(x,t_0) = m_* + \Phi(x,0)\left[\exp(\beta v\int_0^{t_0} c^0(\tau)d\tau) + \beta\int_0^x \Phi(y,0)dy\right]^{-1} \tag{4.2}$$

From the differential relation between the porosity and the concentration given in Section 3, which holds on any characteristic, we also obtain from condition (4.1) that on the curve $x = \xi(t, t_0)$

$$c(x,t_0) = \left[1 + \left(\frac{1}{c^0(t_0)} - 1\right)\exp\left(\beta\int_0^x (m - m_*)dy\right)\right]^{-1} \tag{4.3}$$

Finally, the characteristic itself $x = \xi(t, t_0)$ is defined by the equation

$$v(t - t_0) = \int_0^x m(y,t_0)dy \tag{4.4}$$

which also completes the construction of the required solution.

Omitting the calculations, we note the following properties of the solution obtained, which hold for the natural constraints on the initial data (3.1) indicated in Section 3, and under the condition that in (4.1) $0 < c^0(t) < 1$. Firstly, Eq. (4.4) is solvable for t_0. Secondly, the curve defined by this equation when $t_0 = 0$ coincides with the curve $x = \xi(t, 0)$ from (3.5). Hence solution (4.2)–(4.4), (3.3) is defined everywhere in ω_-. In this region, for continuous $m_0(x)$ and $c^0(t)$ it is continuous, exists and the derivative $\partial m/\partial t$ is continuous. For smooth $c^0(t)$ the function $c(x, t)$ is also continuously differentiable in ω_-. If $c^0(t)$ suffers a discontinuity for a certain $t = t_*$, it will be continued along the characteristic emerging from the point $(0, t_*)$. When $c^0(0) \neq c_0(0)$ a discontinuity occurs in the concentration on passing through the characteristic $x = \xi(t, 0)$ separating the regions ω_1 and ω_+. Finally, if there is a sudden jump in the porosity on the x axis at $x = x_*$, it will be present for all $0 < t$ at points (x_*, t). In this case, the solution is initially constructed in the strip $\{0 < x < x_*, 0 < t\}$, from which the values $c(x_*, t)$ are determined, it is substituted into conditions (4.1) and the solution is then found in the quadrant $\{x_* < x, 0 < t\}$.

5. COMPARISON WITH EXPERIMENT

As an example we will consider the problem of the penetration of a suspension with a specified constant intensity into an initially pure porous sample of length L. In this case $m(x, 0) = m_0 = \text{const}$, $c(0, t) = c^0 = \text{const}$ and $v = \text{const}$. Since $c(x, 0) = c_0 \equiv 0$, by virtue of (3.2)–(3.4) everywhere ahead of the penetration front $c(x, t) = 0$ and $m(x, t) = m_0$. The characteristics of the first equation of system (1.7) are straight lines, and the position of the front is defined by the equation $x = vt/m_0$.

It can be shown that, for the case considered, representation (4.2)–(4.4), (3.3) of the solution in the region $\{m_0 x < vt, 0 < t\}$ reduces to the form

$$m = m_* + s(m_0 - m_*), \quad c = c^0(1 - as)/(1 - c^0 as) \tag{5.1}$$

The function $s(x, t)$ is found from the equation

$$\begin{aligned} &F(s; a, b) = s^{1/(1-c^0)} - (1 - as)b = 0, \quad 0 < s < 1 \\ &a = 1 - \exp[-\beta(m_0 - m_*)x] \\ &b = \exp[\beta(m_0 - m_*)x - \beta c^0(\upsilon t - m_0 x)/(1 - c^0)] \end{aligned} \tag{5.2}$$

The unique solvability of the latter follows from the monotonicity of F with respect to s and the inequalities $F(0; a, b) = -b < 0$, $0 < F(1; a, b) = 1-(1-a)b$, since on the front $m_0 x < \upsilon t$.

We will show that the solutions derived in Section 2 are asymptotic. To do this we write Eq. (5.2) in the form

$$s(1 - as)^{c^0 - 1} = \exp\left\{\beta(m_0 - m_* + c^0 m_*)\left[x - \frac{c^0 \upsilon t}{m_0 - m_* + c^0 m_*}\right]\right\}$$

Its right-hand side and the right-hand side of Eq. (2.5) will be equal if we put $m^* = m_0$, $c_* = c^0$ and define the velocity of the wave σ from (2.3). With this choice, the definitions (2.4) and (5.1) for the function s are identical. The left-hand sides of these equations will differ only in the factor a: in (2.5) $a = 1$ and in (5.2) it depends on x. But at a sufficient distance from the entry of the sample $a \approx 1$, and here not only will the values of s defined by Eqs (2.5) and (5.2) be the same, but also expressions (2.6) and (5.1) for the concentration. This is also the reason why the flow reaches a simple-wave mode, as has been observed in a number of experiments.

Further, we will use the dimensionless parameters m_*/m_0, $\beta m_0 L$, $\sigma m_0/\upsilon$ and the variables x/L, $(\upsilon t)/(L m_0)$, m/m_0, c/c^0, which we again denote by m_*, β, σ and x, t, m and c, respectively. After changing to these we will have $m_0 = 1$ and $\upsilon = 1$.

In Fig. 1(a) we show the changes with time of the distribution along the longitudinal coordinate x of the colmataged fraction of the pore space $P = 1 - s$ for $\upsilon \approx 0.5$ m/hour and $c^0 = 0.0007$; fore the lower curve $t = 5$, for the middle curve $t = 15$ and for the upper curve $t = 25$. The small circles in Fig. 1 are experimental data [10]. The parameters β and m_* were equal to 26.0 and 0.7, respectively. The effect of the input concentration c^0 on $P(x)$ is shown in Fig. 1(b) for the same values of v, β and m_* as above and $t = 10$; for the lower curve $c^0 = 0.0007$ and for the upper curve $c^0 = 0.0014$. In Fig. 1(c) we show graphs of $P(x)$ for $t = 10$, $c^0 = 0.0007$ and the previous values of β and m_*, but for two rates of input of the suspension: $\upsilon \approx 0.5$ m/hour for the lower curve and 1.4 times greater for the upper curve.

Comparison with experiment shows that the model employed describes the distribution of the volume of particles captured from the flow over space and time quite well. It also correctly takes into account the input concentration. It can be seen that the seepage rate is the important parameter, while the linear dependence on it used here does not only have a theoretical meaning. The linearized approximation [8] does not give such agreement with experiment [10].

The results of experiment [10] relate to those times when the simple-wave zone, described by representations (2.1)–(2.6), has not yet sufficiently developed. For such regimes, as noted in Section 2, a characteristic feature is the presence of inflection points. In Fig. 2 we show two groups of curves, calculated from (5.1) and (5.2). The first

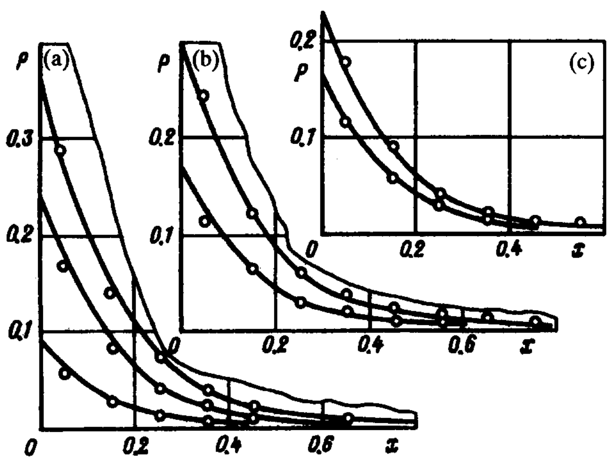

Fig. 1.

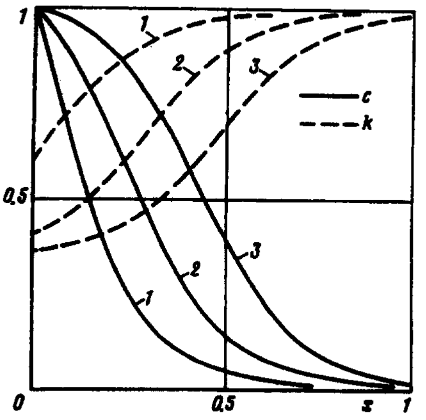

Fig. 2.

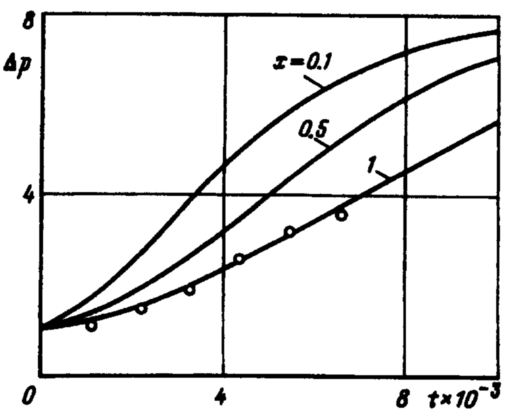

Fig. 3.

of these (the continuous curves) describes the changes in the reduced concentration in the flow direction, and the second (the dashed curves) represents the permeabilities. Here $\upsilon \approx 0.5$ m/hour, $\beta = 26, m_* = 0.7$ and $c^0 = 0.0007$. We chose as the values of the dimensionless time the time at which an inflection point occurs in the concentration profile at the input to the sample, i.e. $t = 38$ (curves 1), and the multiples of this: $t = 114$ (curves 2) and $t = 190$ (curves 3). The experimental curves behave in a similar way, when the times taken for the simple-wave zone to develop are reached.

The pressure drop at the input and at the point x relates essentially to the corresponding value of the pressure drop in the motion of the pure fluid, i.e. to $p(0, 0) - p(x, 0) = \mu\upsilon k_0^{-1}x$. Taking relations (1.5), (1.6) and (5.1) into account, in dimensionless variables this relative pressure drop can be written in the form

$$\Delta p(x,t) = \int_0^1 \frac{dy}{[m_* + (1-m_*)s(xy,t)]^n}$$

In Fig. 3, for $n = 3$ and $c^0 = 0.00005$, we show Δp as a function of the dimensionless time for various values of x. The points indicate experimental data [11, 1] for the case of the total pressure drop. Here β and m_*, obtained by comparing the calculated and experimental concentration profiles, were equal to 9 and 0.5, respectively. Hence, the model also qualitatively correctly describes the pressure dynamics during colmatage.

In conclusion, we point out the following. If the input concentrations c^0 are sufficiently small, then, up to a certain instant of time, behind the front the concentration profile is simply shifted along the flow. The changes in the parameter b with time from (5.2), related to this, only have an effect when the suspension reaches the exit from the sample. During the following instants a good approximation to relations (5.1) and (5.2) is the relation obtained previously [1] for the case $m_* = 0$.

$$c = c^0\{1 + [\exp(\beta(1 - m_*)x) - 1]\exp(-\beta c^0 t)\}^{-1}$$

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